Mixed Birth-death and death-Birth processes in structured populations

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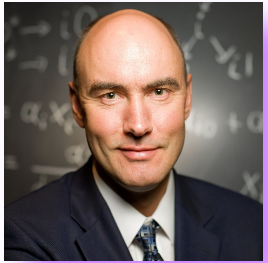

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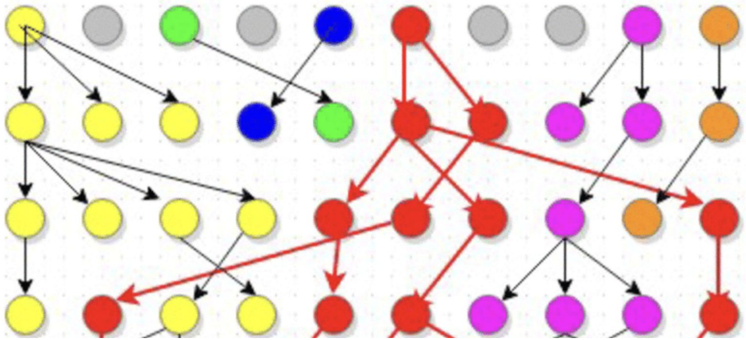
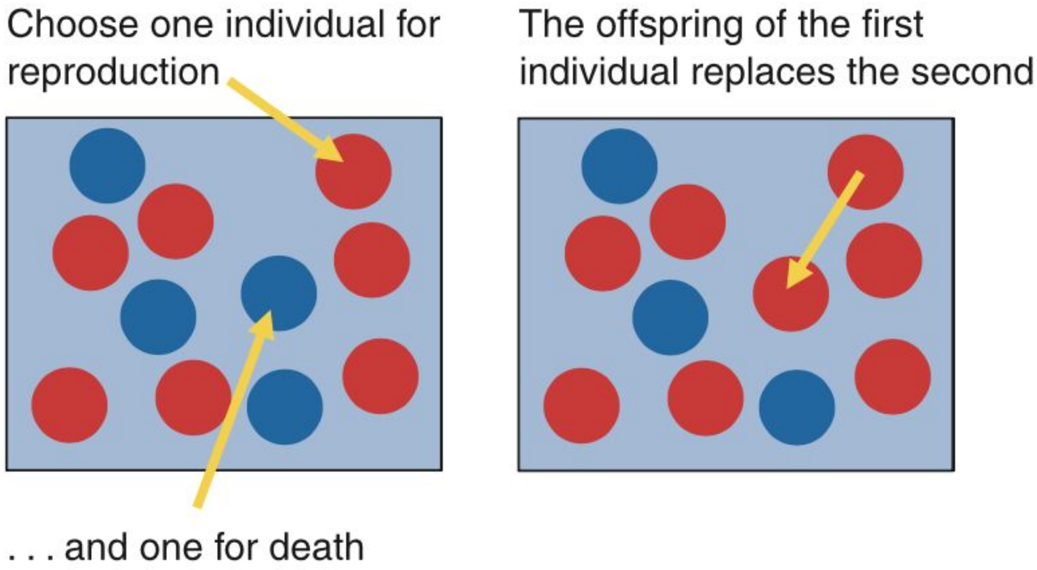
Drift models in population genetics

Wright-Fisher



Moran



Moran process

N individuals

Two types: A & B

Type A has fitness 1

Type B has fitness r

At each discrete time step:

1. choose random individual for birth proportional to fitness
2. choose random individual for death uniformly
3. Individual 1 places an offspring in the population while individual 2 is removed from the population

Interactive simulation

Moran fixation probability

$$\frac{1}{N}$$

or

$$\frac{1 - 1/r}{1 - 1/r^N}$$

if r = 1

if r ≠ 1

Moran fixation time

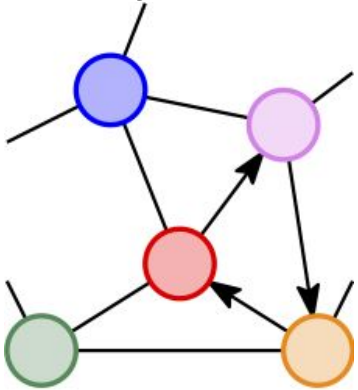
$\Theta(N^2)$ or $\Theta(N \log N)$

if r = 1

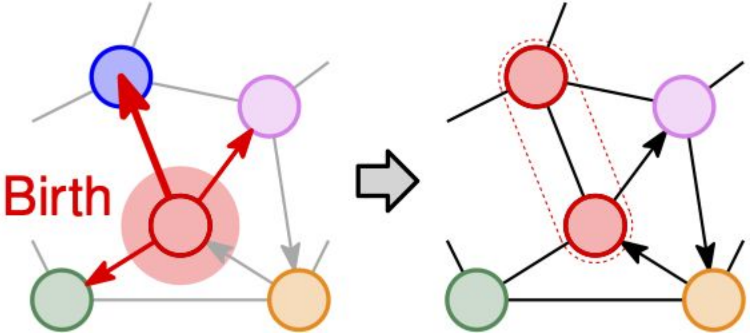
if r ≠ 1

BUT… all populations have structure!

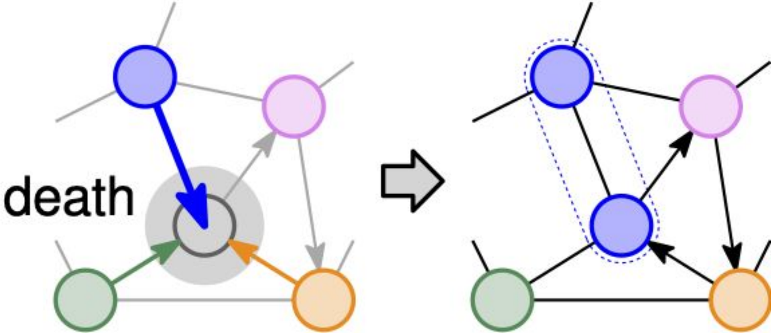
Evolutionary Graph Theory



birth-death (bd)



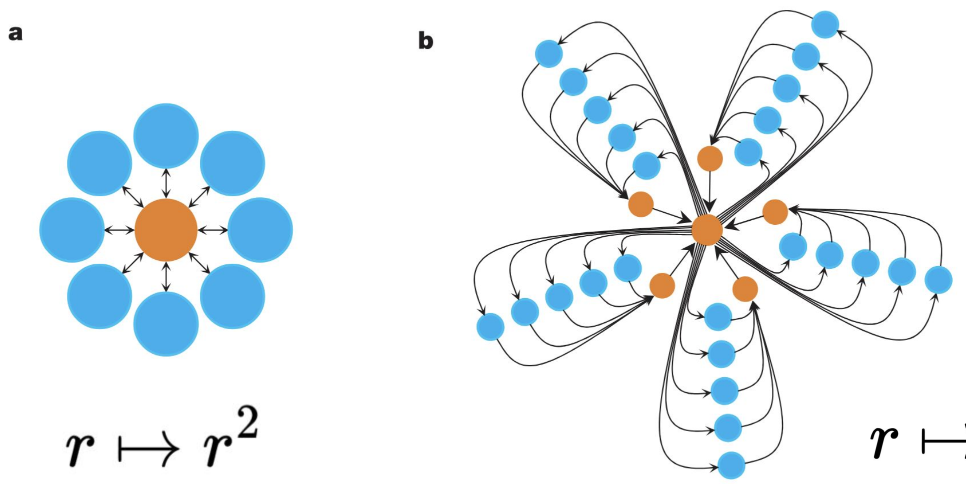
death-birth (db)



birth-death

$$p_{\text{fix}}(u) \propto 1/\deg(u)$$

Amplifiers of selection



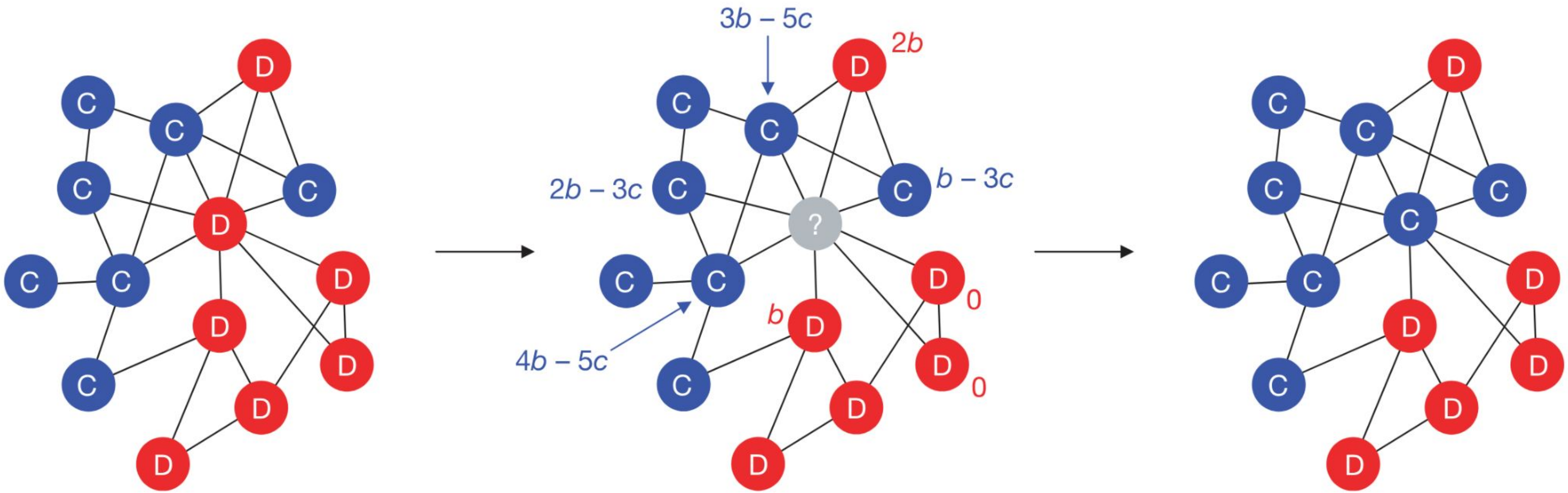
$r \mapsto r^2$

$r \mapsto r^k$

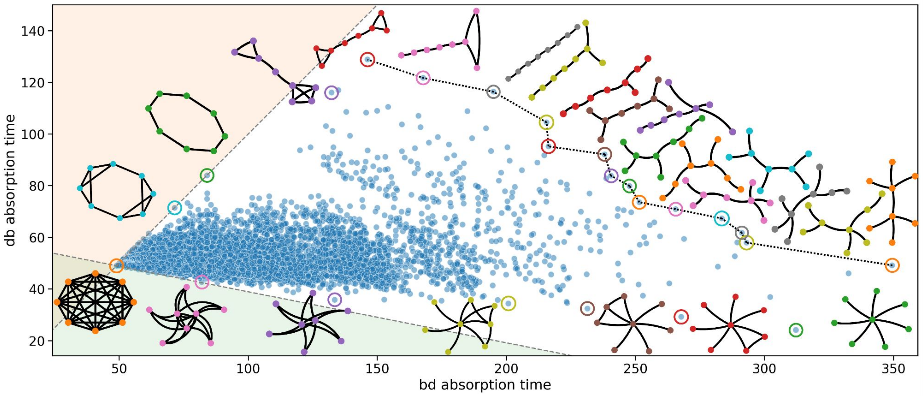
death-birth

$$p_{\text{fix}}(u) \propto \deg(u)$$

Iterated prisoner's dilemma



Tensions between Birth-death vs death-Birth



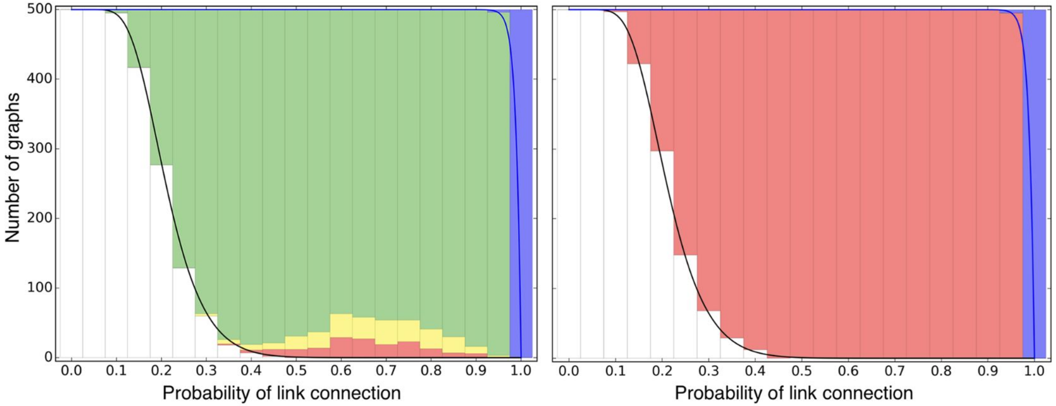
Bd updating can never favor cooperation

Structures that maintain diversity are sensitive to update order

Amplifiers vs suppressors

Birth-death

death-Birth



N = 14

Mixed updating

N individuals

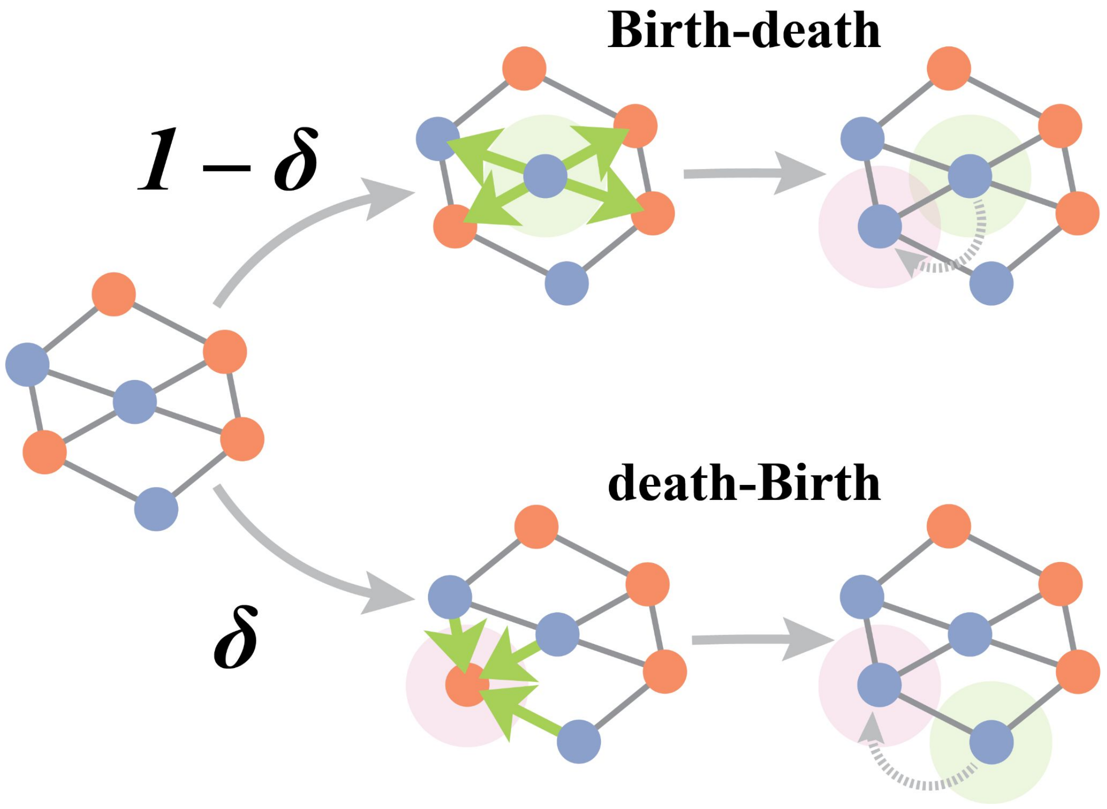
Two types: A & B

Type A has fitness 1

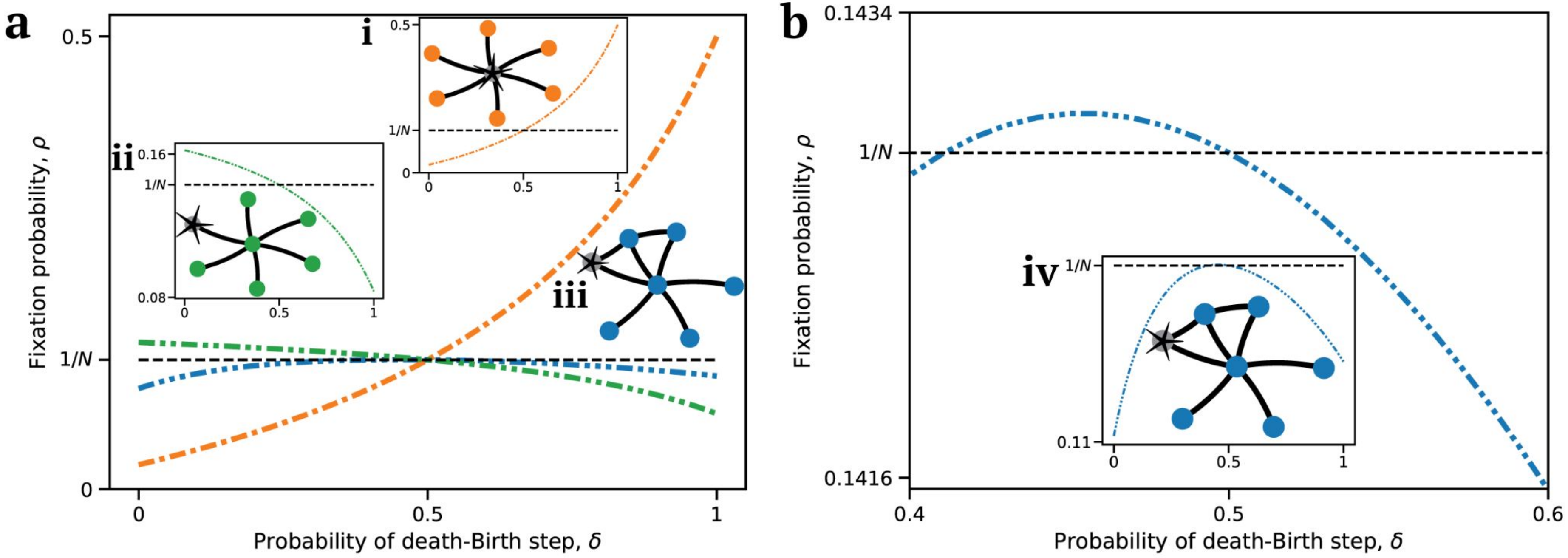
Type B has fitness r

At each discrete time step:

0. Choose dB step with prob δ and Bd step with prob 1-δ
1. choose random individual for birth proportional to fitness
2. choose random individual for death uniformly
3. Individual 1 places an offspring in the population while individual 2 is removed from the population


Birth-death
1 – δ
death-Birth
δ

Increasing, decreasing, or non-monotone fixation prob (fp)!



Some initial results for neutral evolution, fix prob

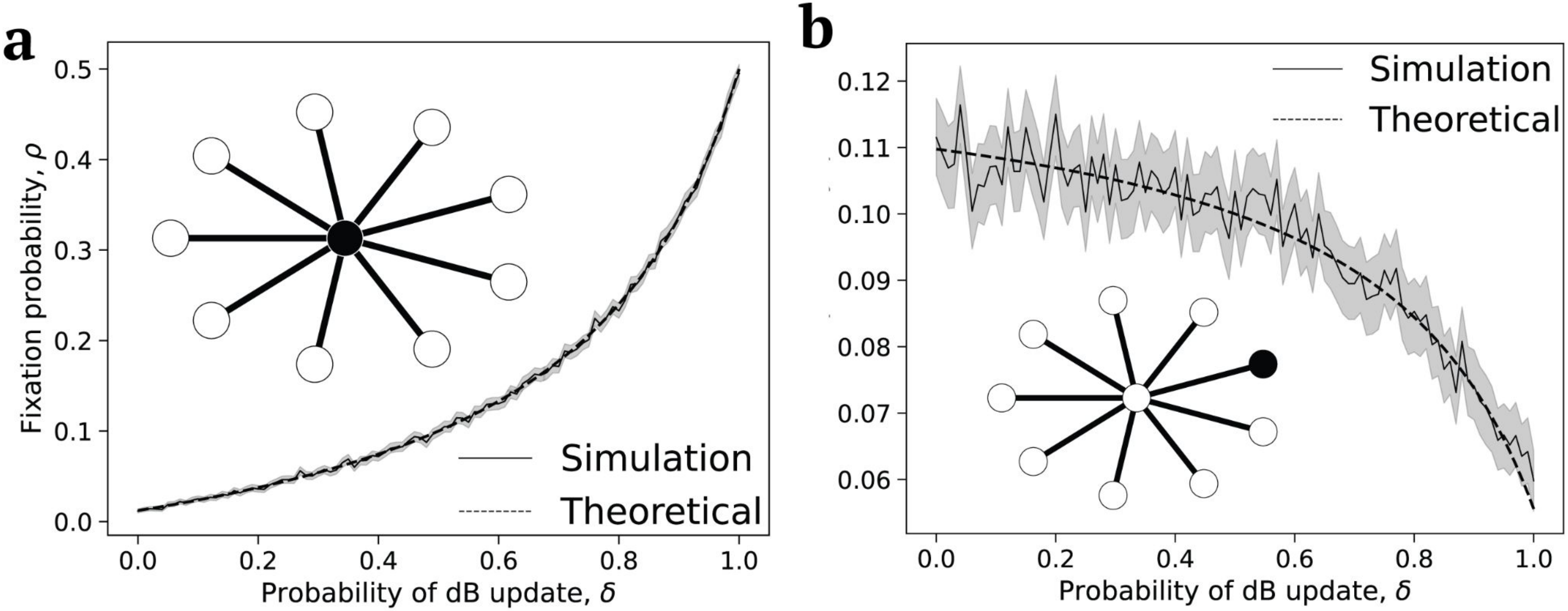
1. fp = 1/N for all graphs, all δ, r=1, uniform initialization

2. fp = 1/N for all graphs, δ = ½, r=1, any initial mutant location

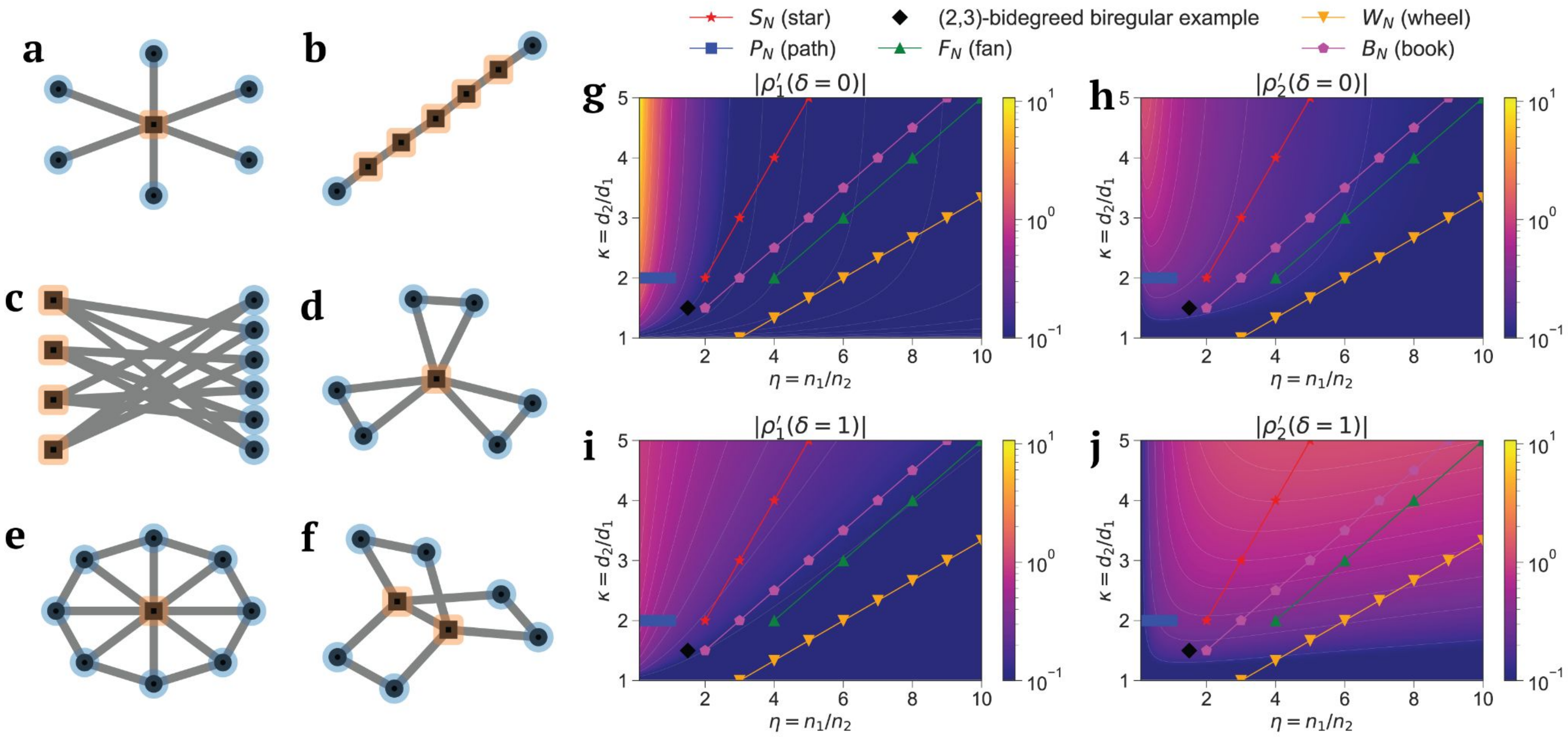
3. Impossible for both dB and Bd fp to be larger than fp when δ = ½, for all graphs, r=1, any initial mutant location

4. fp = 1/N for regular graphs, all δ, r=1, any initial mutant location

Star neutral evolution



Bidegreed graphs, sensitivity



Bidegreed graphs, sensitivity

bidegreed graph, $d_1 < d_2$, $r=1$:

mutant starts on <u>d1</u> vertex: fp is strictly <u>decreasing</u> in δ

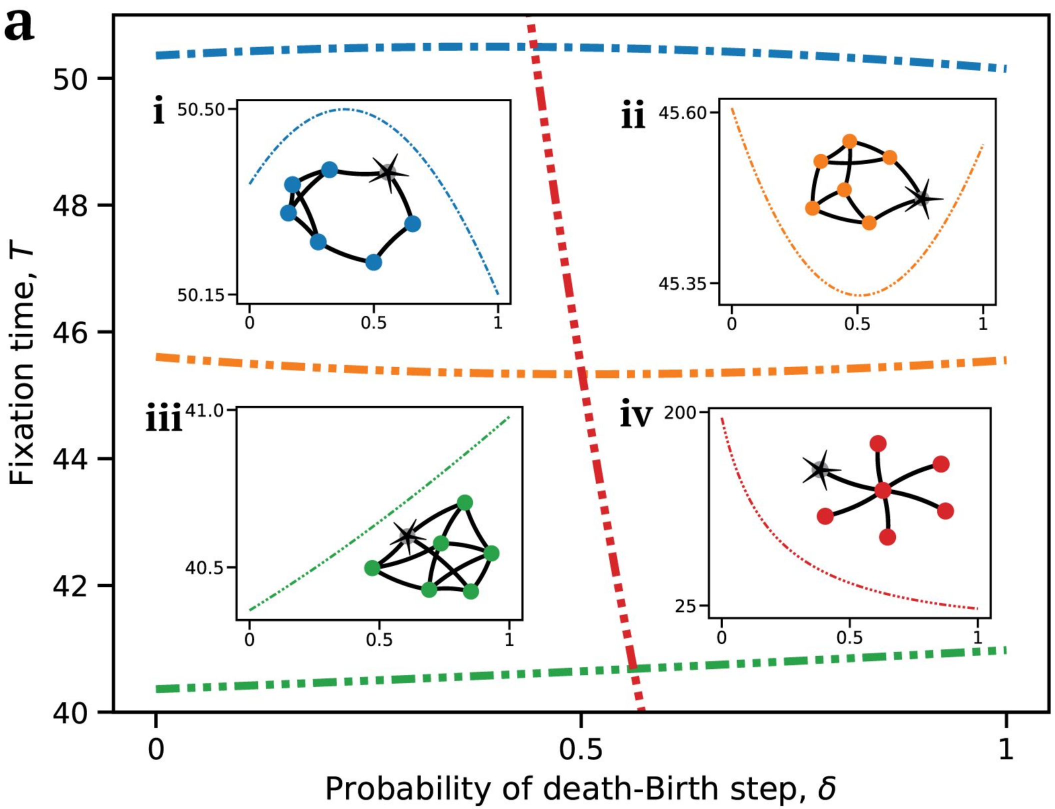
mutant starts on <u>d2</u> vertex: fp is strictly <u>increasing</u> in δ

fp when starting on <u>d2</u> vertex is proportional to

$$\frac{(1-\delta)d_1+\delta d_2}{(1-\delta)d_2+\delta d_1}$$

otherwise, proportional to 1

Increasing, decreasing, or non-monotone fixation time (ft)!

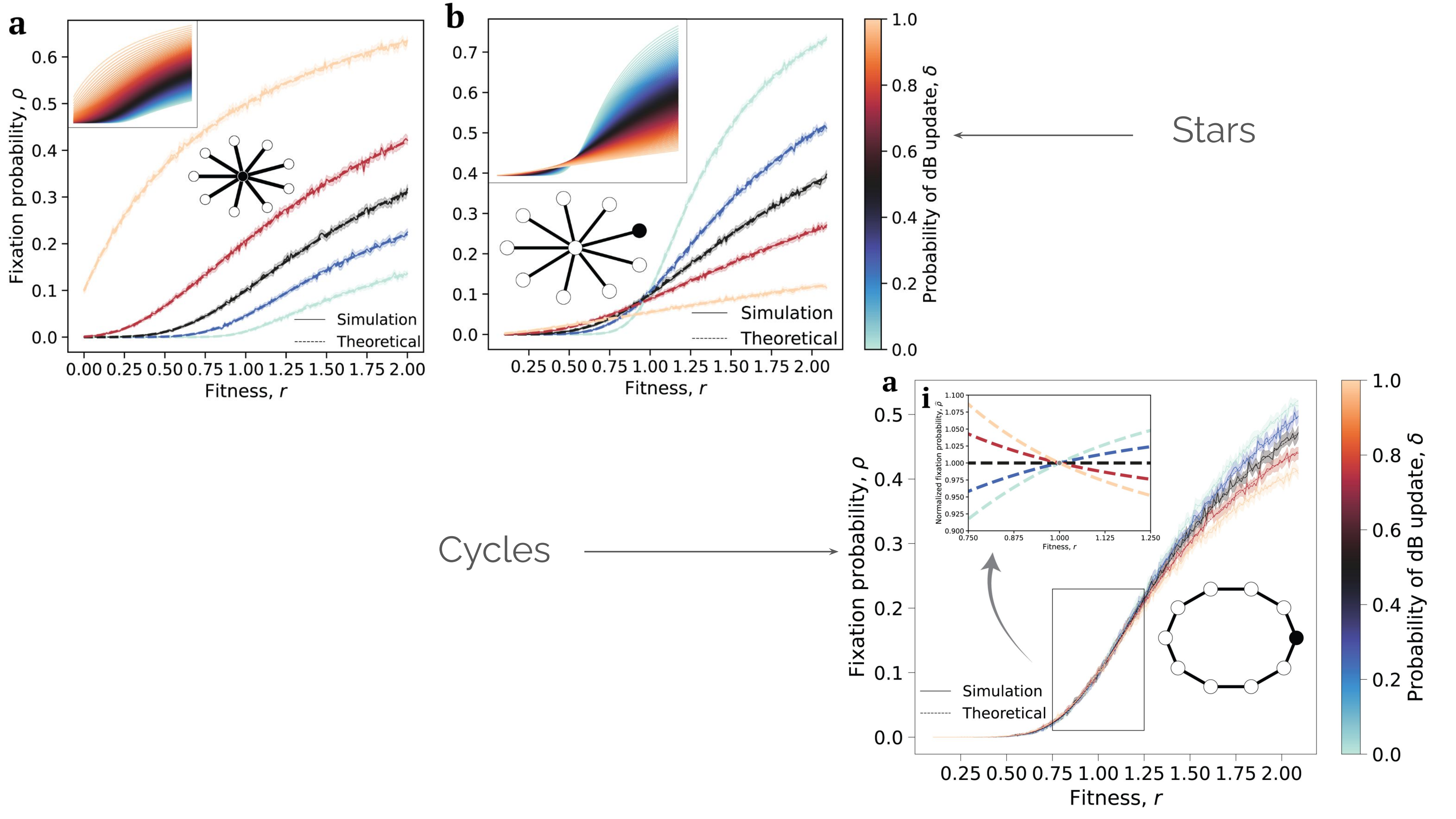


Some initial results for neutral evolution, fix time

1. ft < $N^5/4$ for <u>all graphs</u>, **δ** = ½, r=1, any initial mutant location

2. ft < $N^4 (d2/d1)^6/4$ for <u>bidegreed graphs</u>, **δ** = ½, r=1, any initial mutant location

… what about non-neutral selection?



a
b
Fixation probability, ρ
Fitness, r
Simulation
Theoretical
Probability of dB update, δ
Stars
Cycles
i
Normalized fixation probability, ρ̃

Efficient estimation algorithms

1. FPRAS (estimation algorithm) for fp for all graphs, **δ** = ½, r > 1

2. FPRAS (estimation algorithm) for fp for <u>almost-regular graphs</u>, all **δ**, r>$(D/d)^2$

max degree

min degree

That's all Folks!