Maintaining diversity in structured populations

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Motivation

Evolution occurs in reproducing populations. The structure of a population affects the time scale and outcome of evolutionary processes. The propensity of populations to maintain diversity is of great interest in evolutionary biology and ecology. Here we calculate how long various population structures can maintain diversity under neutral evolution. In this setting, diversity is lost by random drift.

Model

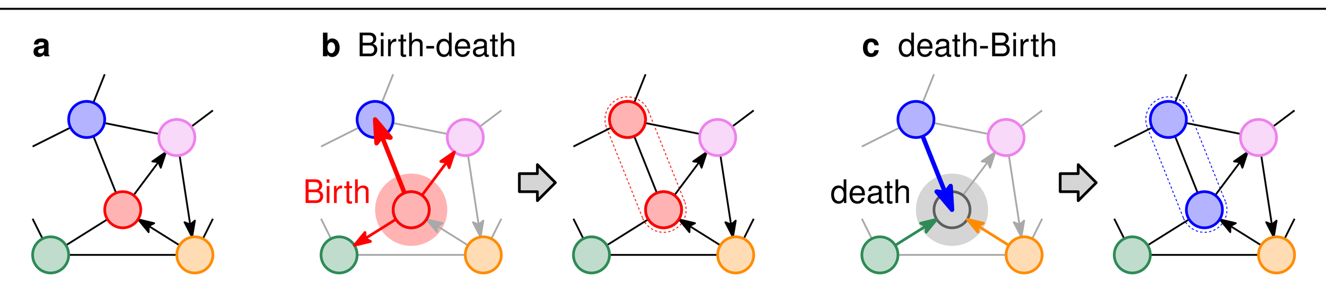

Figure 1. Consider a population of N individuals. Every time step, one individual is selected for birth and one individual for death. The individual selected for death is removed from the population. The individual selected for birth creates a copy of itself at the location of the individual that was selected for death.

Goal

Starting from maximum diversity, we calculate the time until the population becomes homogeneous. We are interested in the expected absorption time of various population structures.

Results

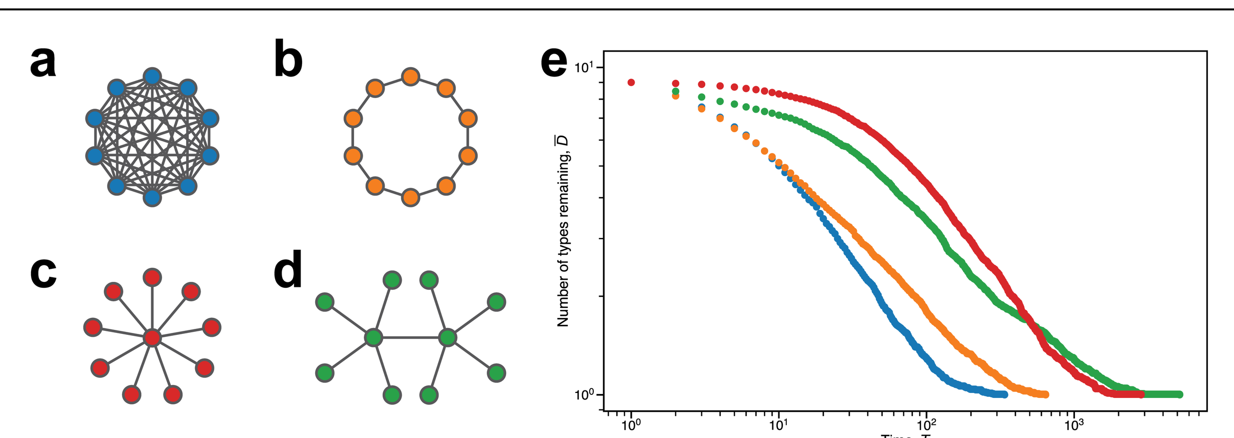

Figure 2. Results of average number of types remaining in the population, $\overline{D}$, at time T, averaged over 250 simulations of birth-death updating per graph.

Results (con't)

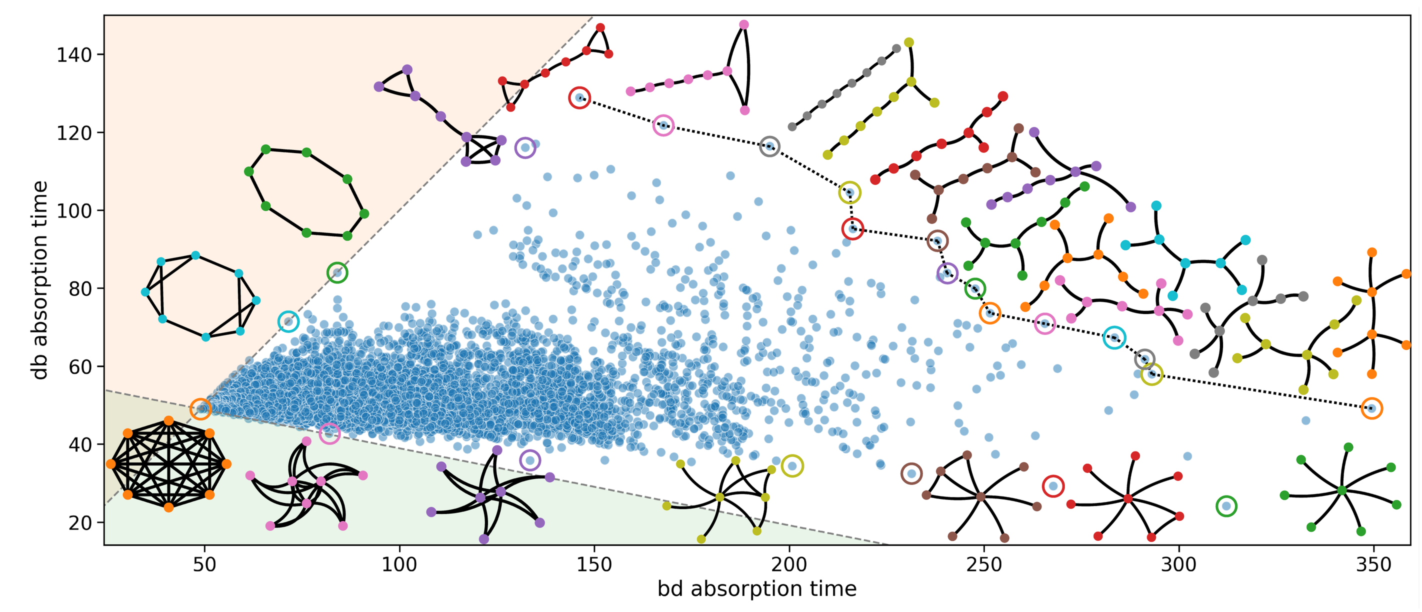

Figure 3. Absorption times for bd vs db updating for all 11117 connected undirected graphs with $N = 8$ vertices. Each blue dot is a graph. The Pareto front (the dashed black lines) connects the two.

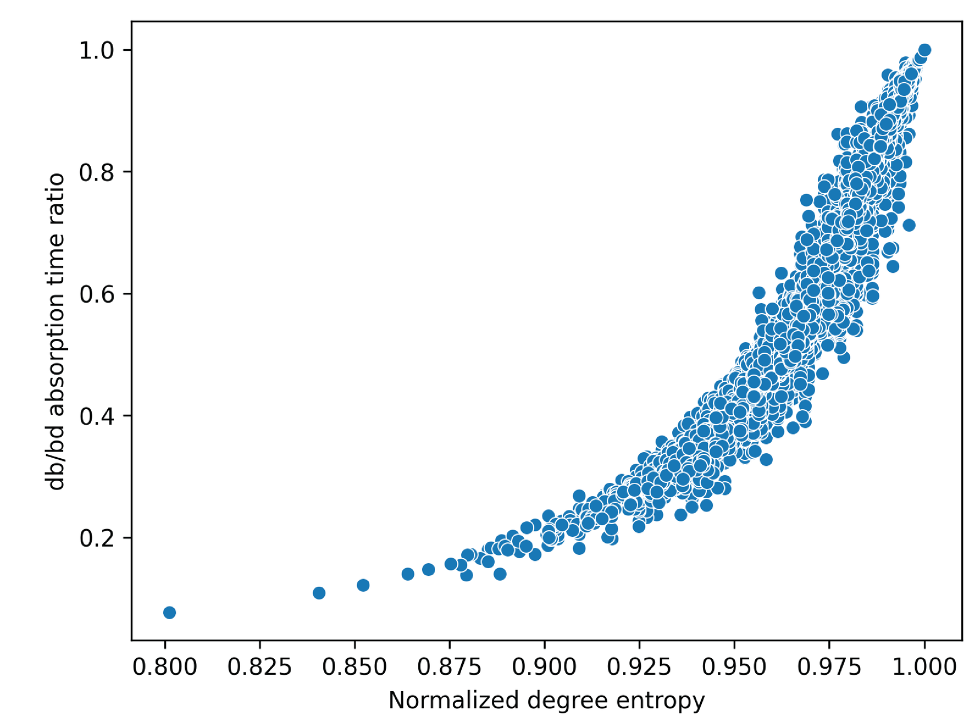

Figure 4. db absorption time divided by bd absorption time versus normalized degree entropy for all graphs with $N = 8$ vertices.

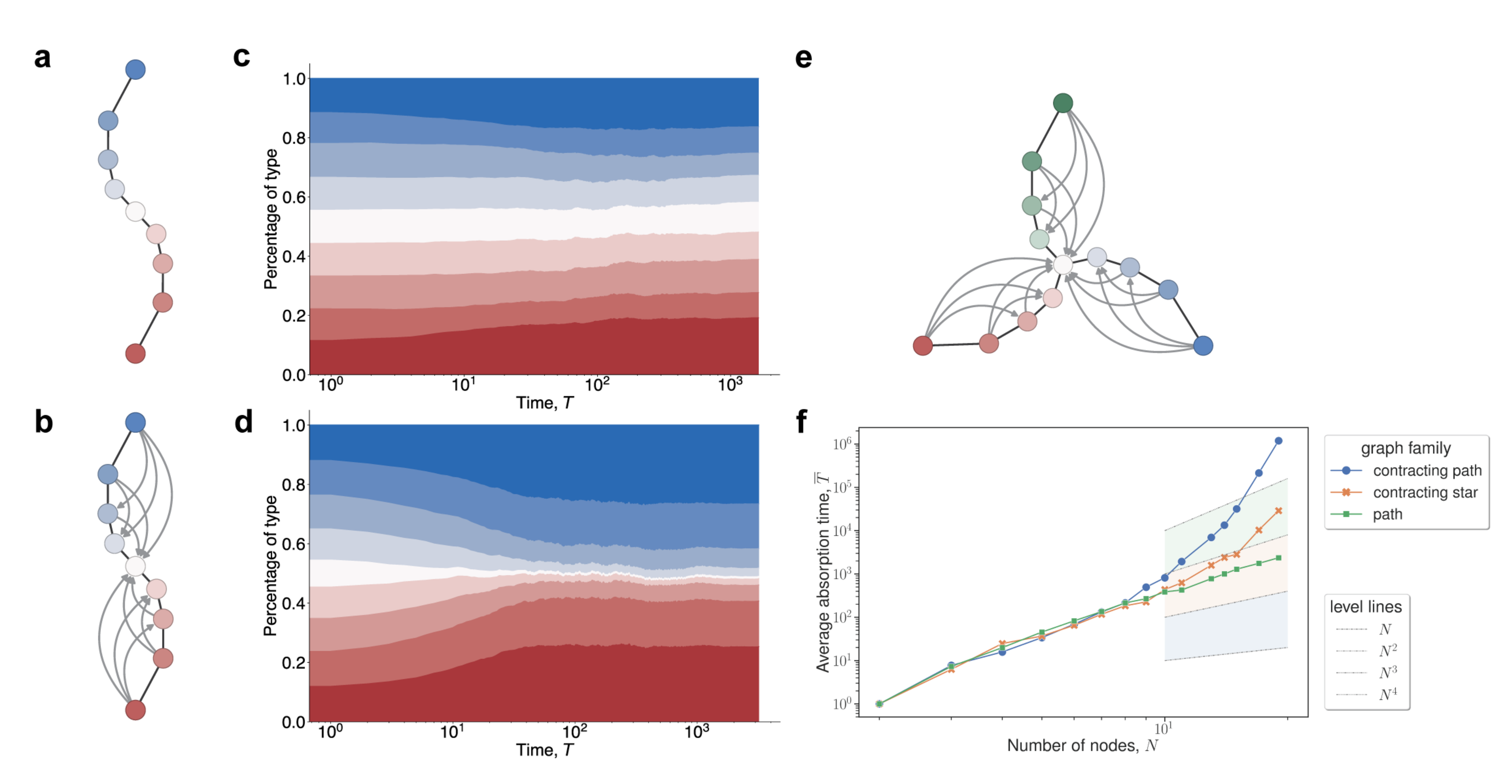

Figure 5. Simulation results of the expected absorption time for birth-death updating of a undirected path, a contracting path, and a contracting star over varying population sizes.

Discussion

	Graph family	bd time	db time
Undirected	Complete	$\Theta(N^2)$	$\Theta(N^2)$
	Cycle	$\Theta(N^3)$	$\Theta(N^3)$
	Star	$\Theta(N^3)$	$\Theta(N \log N)$
	Double star	$\Theta(N^4)$	–
	Barbell	–	$\Omega(N^4)$
	Undirected graph	$O(N^6 \log N), \Omega(N \log N)$	$O(N^5 \log N), \Omega(N \log N)$
Directed	Contracting star	$2^{\Theta(N \log N)}$	$2^{\Theta(N \log N)}$
	Directed graph	$2^{O(N \log N)}$	$2^{O(N \log N)}, \Omega(N \log N)$

Table 1. Asymptotic absorption times for various graph families under bd and db updating.

Key Observation

Separate communities with weak ties are strong maintainers of diversity. However, the definitions of “communities” and “weak ties” depend on update rules.

Future Work

Add mutations!

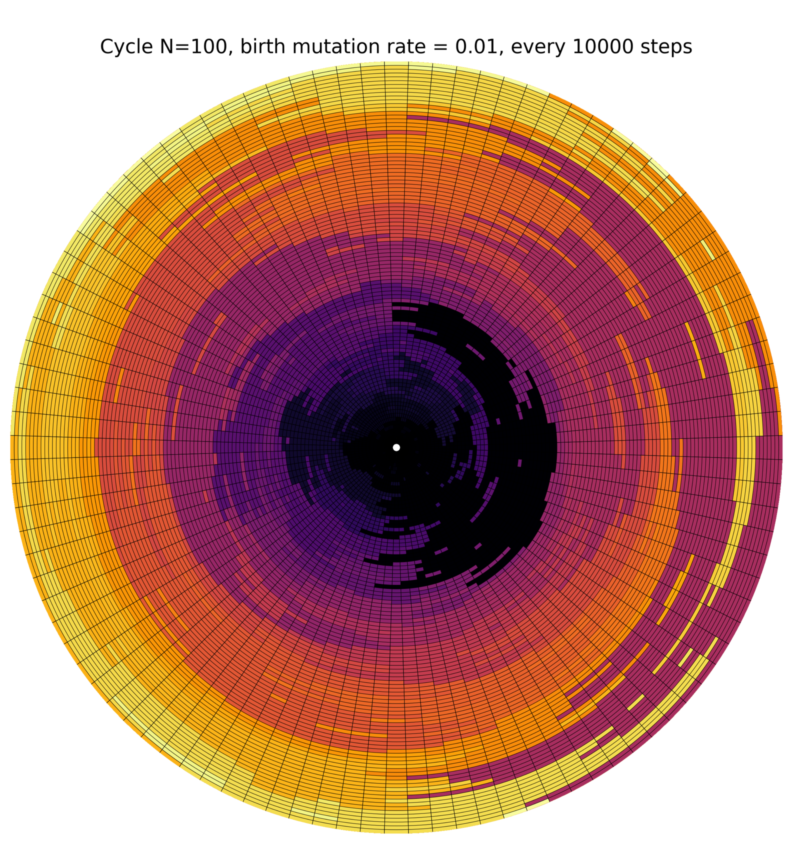

Figure 6. Cycle population birth-death process with infinite-allele mutation model. Initially all individuals are the same type. Snapshots of the population composition over time (radial axis). Lighter colors indicate more recent mutations.